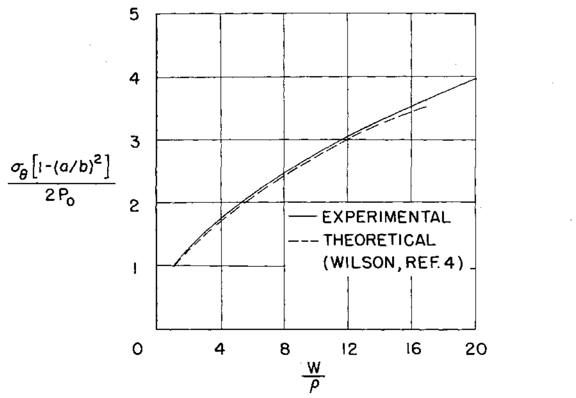

Fig. 3 Comparison of theory and experiment for $N = 4$

and compared to a notched strip. The following relation was deduced:

$$\frac{\sigma_\theta}{P_0} = -\left(\frac{2 - w/b}{w/b}\right)^{1/2} N^{-1/3} \times \left[1 + 2\left\{\frac{W}{\rho}\left(\frac{1 - w/b}{w/b}\right)\right\}^{1/2}\right] \quad (2)$$

This expression fits the data quite well except in the region near the limit points as $w/\rho \rightarrow 0$.

In summary, the results of these tests indicate that the maximum tangential stress at the star point can be reduced in the following ways: 1) increasing the fillet radius, 2) increasing the web fraction, and 3) increasing the number of star points. It also has been shown that major geometrical changes in the inverse star-point region have little effect on the maximum stress. However, one should not conclude that the maximum stress is unaffected by small geometrical changes in the star-point region. Recent tests have shown that relatively minor changes in the star-point contour can reduce the maximum stress by more than a factor of 3.

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A Simple MHD Flow with Hall Effect

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In magnetohydrodynamics, the Hall effect rotates the current vector away from the direction of the electric field and generally reduces the level of the force that the magnetic field exerts on the flow. It usually is measured by the parameter $\omega\tau$, where $\omega = eB/m$ is the angular velocity of the electron orbits around the field lines, and τ is the mean time between scattering collisions for the electrons. The form of Ohm's law which accounts for the Hall effect is[1]

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - (e\tau/m)\mathbf{j} \times \mathbf{B} \quad (1)$$

This note will describe a simple flow in which the Hall currents can be calculated exactly and the results compared with those that follow from the usual simplifying assumptions of reducing in fixed ratios the conductivities parallel and perpendicular to the field lines.

Consider a flow of the type illustrated in Fig. 1. A wire carrying current I coincides with the z axis, and the magnetic Reynolds number is assumed small so that (in cylindrical coordinates)

$$\mathbf{B} = (0, \mu_0 I/2\pi r, 0) \quad (2)$$

The flow is assumed to be given, and the velocity vector is restricted to be of the form

$$\mathbf{v} = U\{[-1 + (a^2/r^2)]\cos\theta, [1 + (a^2/r^2)]\sin\theta, 0\} \quad (3)$$

which represents uniform two-dimensional flow without circulation past a cylinder of radius a, with velocity U at infinity. In the absence of the Hall effect, the current is in the z direction and is given by

$$j_z = (-\sigma U \mu_0 I/2\pi r)[1 - (a^2/r^2)]\cos\theta$$
$$= -(N_e eU)(\omega\tau)[1 - (a^2/r^2)]\cos\theta \quad (4)$$

where $\sigma = N_e e^2\tau/m$, N_e being the electron density. However, the Hall effect will reduce this current and also produce a current pattern in the plane of the flow. A nondimensional distance is defined by

$$\bar{r} = (\omega\tau)^{-1} = 2\pi m r/\mu_0 e\tau I \quad (5)$$

and let $r = a$ correspond to $\bar{r} = \bar{a}$. Since the current vector

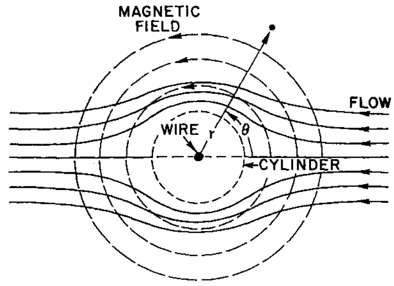

Fig. 1 Illustration of problem studied. The flow past the cylinder is unperturbed, and the current pattern is to be determined

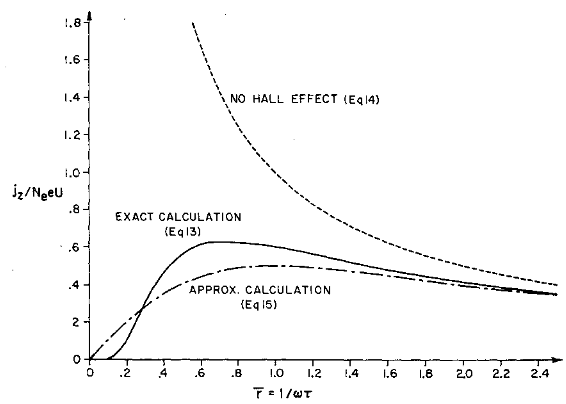

Fig. 2 Illustration of the reduction in the conduction current due to the Hall effect as calculated exactly and approximately

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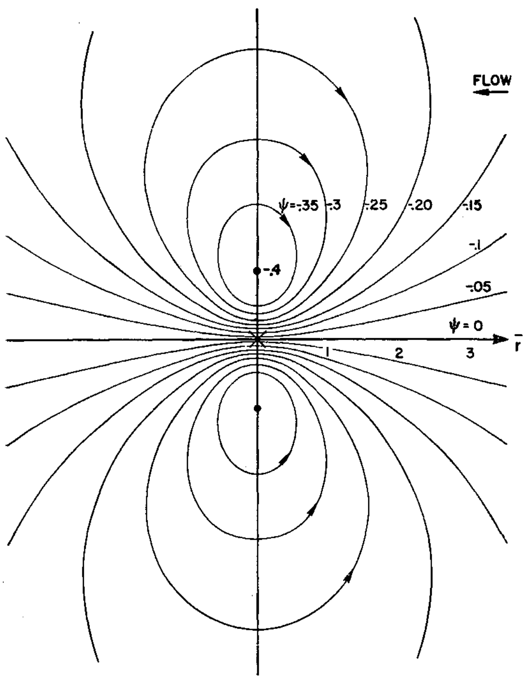

Fig. 3 Illustration of the current pattern induced in the plane of the flow by the Hall effect

has zero divergence, it may be written

$$\mathbf{j}/N_e eU = (\partial\psi/\bar{r}\partial\theta, \quad -\partial\psi/\partial\bar{r}, \quad j_z/N_e eU) \tag{6}$$

Furthermore, the electric field vector is irrotational and lies in the plane of the flow. Thus, it may be derived from a potential $\phi(\bar{r}, \theta)$:

$$\mathbf{E}e\tau/Um = (\partial\phi/\partial\bar{r}, \quad \partial\phi/\bar{r}\partial\theta, \quad 0) \tag{7}$$

Substituting these expressions for **j**, **E**, **v**, and **B** into the Ohm's law, one finds

$$\begin{aligned} \partial\psi/\partial\theta &= -\bar{r}\partial\phi/\partial\bar{r} + j_z/N_e eU \\ \partial\psi/\partial\bar{r} &= -\partial\phi/\bar{r}\partial\theta \\ j_z/N_e eU &= -(1/\bar{r})[1 - (\bar{a}^2/\bar{r}^2)]\cos\theta - (1/\bar{r}^2)(\partial\psi/\partial\theta) \end{aligned} \tag{8}$$

From these equations it is clear that one can find a solution of the form

$$\begin{aligned} \psi(\bar{r}, \theta) &= G(\bar{r})\sin\theta \\ \phi(\bar{r}, \theta) &= F(\bar{r})\cos\theta \\ j_z/N_e eU &= J(\bar{r})\cos\theta \end{aligned} \tag{9}$$

The equation for G which results is

$$\bar{r}(d/d\bar{r})[\bar{r}(dG/d\bar{r})] - [1 + (1/\bar{r}^2)]G = (1/\bar{r})[1 - (\bar{a}^2/\bar{r}^2)] \tag{10}$$

The general solution of this equation which tends to zero as $\bar{r} \to \infty$ is

$$G(\bar{r}) = AI_1(1/\bar{r}) + K_1(1/\bar{r}) - \bar{r} + \bar{a}^2/\bar{r} \tag{11}$$

where A is an arbitrary constant and I_p and K_p are the modified Bessel functions of the first and second kinds of order p as defined, say, by Watson.[2] The corresponding solutions for $F(\bar{r})$ and $J(\bar{r})$ are

$$\begin{aligned} F(\bar{r}) &= [-AI_1'(1/\bar{r}) - K_1'(1/\bar{r}) - \bar{r}^2 - \bar{a}^2]/\bar{r} \\ J(\bar{r}) &= [-AI_1(1/\bar{r}) - K_1(1/\bar{r})]/\bar{r}^2 \end{aligned} \tag{12}$$

The boundary condition that determines A normally would be applied on the surface of the cylinder $\bar{r} = \bar{a}$. If the cylinder is a conductor, it is an equipotential and $F(\bar{a}) = 0$. If it is an insulator, the radial component of the current vanishes, so that $G(\bar{a}) = 0$. For the sake of brevity, only the case $\bar{a} = 0$ corresponding to a uniform unperturbed flow will be considered. The appropriate boundary condition at $\bar{r} = 0$ is then $G(0)$ finite, so that $A = 0$. The current in the z direction then is given by

$$j_z/N_e eU = -(\omega\tau)^2 K_1(\omega\tau)\cos\theta \tag{13}$$

With no cylinder present, the current in the absence of Hall effect reduces from Eq. (4) to

$$j_z/N_e eU = -\omega\tau\cos\theta \tag{14}$$

Elementary discussions of the Hall effect usually state that the conductivity normal to the field lines is reduced by the factor $[1 + (\omega\tau)^2]^{-1}$. This result ignores the electric field induced by the Hall effect which depends on the geometry, but if it is applied to this case, one finds

$$j_z/N_e eU = -\omega\tau[1 + (\omega\tau)^2]^{-1}\cos\theta \tag{15}$$

Equations (13–15) are plotted in Fig. 2. It is seen that j_z is reduced substantially by the Hall effect when $\omega\tau$ is of the order of unity, and that Eq. (15) gives a reasonably accurate estimate of j_z except where $\omega\tau$ is large. In this region the induced electric field cannot be ignored. The current lines in the plane are given by

$$\psi = [K_1(\omega\tau) - (\omega\tau)^{-1}]\sin\theta \tag{16}$$

and these are shown in Fig. 3. The equipotentials are given by

$$\phi = [\omega\tau K_0(\omega\tau) + K_1(\omega\tau) - (\omega\tau)^{-1}]\cos\theta \tag{17}$$

Note that the cylinder defined by $\omega\tau \approx 1.1$ is an equipotential.

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Propagation of Thermal Disturbances in Rarefied Gas Flows

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Grad and Ai recently have shown that, when the flow time is much less than the average collision time or relaxation time in a rarefied gas field, the one-dimensional linearized Grad equations indicate the propagation of disturbances along two distinct characteristics.[1, 2] The phenomena of propagation of signals along two distinct wave fronts in the rarefied gas field are strikingly similar to the propagation phenomena observed for waves of finite amplitude in continuum flows.[3]

The propagation phenomena in the rarefied gas field occur only in the limit of $t/t_f \ll 1$, where t_f is the relaxation time. However, in extremely rarefied gases ($<10^{10}$ particles per unit volume), the disturbance propagation can occur over very large distances. The similarity of the behavior in this limit

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